If two points are no more than one foot apart, you can find the distance between them by using an ordinary ruler. (The inch rulers below have been reduced to fit on the page.)

In the figure, the distance between point A and point B is 5 inches. Of course, there is no need to place the zero of the ruler on point A. In the figure below, the 2-inch mark is on point A. In this case, AB, measured in inches, is $|7 - 2| = |2 - 7| = 5$, as before.

The number obtained as a measure of distance depends on the unit of measure. On many rulers one edge is marked in centimeters. Using the centimeter scale, the distance between the points A and B above is about 12.7 cm.

1. Determine the length of each segment in centimeters.

   \[ \begin{align*}
   D & \quad E & \quad F \\
   a. \ DE = & \quad b. \ EF = & \quad c. \ DF = \\
   \\
   G & \quad H & \quad K \\
   a. \ KH = & \quad b. \ HG = & \quad c. \ GK = \\
   \\
   \text{MATH TERMS} \\
   \text{The Ruler Postulate} \\
   a. To every pair of points there corresponds a unique positive number called the distance between the points. \\
   b. The points on a line can be matched with the real numbers so that the distance between any two points is the absolute value of the difference of their associated numbers.
3. Using your results from Items 1 and 2, describe any patterns that you notice.

4. Given that $N$ is a point between endpoints $M$ and $P$ of line segment $MP$, describe how to determine the length of $MP$, without measuring, if you are given the lengths of $MN$ and $NP$.

5. Use the Segment Addition Postulate and the given information to complete each statement.

   a. If $B$ is between $C$ and $D$, $BC = 10$ in., and $BD = 3$ in., then $CD = \underline{______}$.

   b. If $Q$ is between $R$ and $T$, $RT = 24$ cm, and $QR = 6$ cm, then $QT = \underline{______}$.

   c. If $P$ is between $L$ and $A$, $PL = x + 4$, $PA = 2x - 1$, and $LA = 5x - 3$, then $x = \underline{______}$ and $LA = \underline{______}$.
The **midpoint** of a segment is the point on the segment that divides it into two **congruent** segments. For example, if $B$ is the midpoint of $\overline{AC}$, then $\overline{AB} \cong \overline{BC}$.

![Segment Diagram]

6. Given: $M$ is the midpoint of $\overline{RS}$. Complete each statement.
   
   a. If $RS = 10$, then $SM = \underline{_______}$.
   
   b. If $RM = 12$, then $MS = \underline{_______}$, and $RS = \underline{_______}$.

You can use the definition of midpoint and properties of algebra to determine the length of a segment.

**EXAMPLE 1**

If $Q$ is the midpoint of $\overline{PR}$, $PQ = 4x - 5$, and $QR = 11 + 2x$, determine the length of $\overline{PQ}$.

<table>
<thead>
<tr>
<th>Step</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$PQ \cong QR$</td>
</tr>
<tr>
<td>2.</td>
<td>$PQ = QR$</td>
</tr>
<tr>
<td>3.</td>
<td>$4x - 5 = 11 + 2x$</td>
</tr>
<tr>
<td>4.</td>
<td>$2x - 5 = 11$</td>
</tr>
<tr>
<td>5.</td>
<td>$2x = 16$</td>
</tr>
<tr>
<td>6.</td>
<td>$x = 8$</td>
</tr>
<tr>
<td>7.</td>
<td>$PQ = 4(8) - 5 = 27$</td>
</tr>
</tbody>
</table>

**Math Tip**

Use $\overline{AB}$ when you talk about segment $\overline{AB}$.

Use $\overline{AB}$ when you talk about the measure, or length, of $\overline{AB}$.
EXAMPLE 2

If \( Y \) is the midpoint of \( \overline{WZ} \), \( YZ = x + 3 \) and \( WZ = 3x - 4 \), determine the length of \( \overline{WZ} \).

1. \( WZ = WY + YZ \)
2. \( WY \cong YZ \)
3. \( WY = YZ \)
4. \( WZ = YZ + YZ \)
5. \( 3x - 4 = (x + 3) + (x + 3) \)
6. \( x - 4 = 6 \)
7. \( x = 10 \)
8. \( WZ = 3(10) - 4 = 26 \)

TRY THESE

Given: \( M \) is the midpoint of \( \overline{RS} \). Use the given information to find the missing values.

a. \( RM = x + 3 \) and \( MS = 2x - 1 \)
   \( x = \) _____ and \( RM = \) _____

b. \( RM = x + 6 \) and \( RS = 5x + 3 \)
   \( x = \) _____ and \( SM = \) _____

You measure angles with a protractor. The number of degrees in an angle is called its measure.

7. Determine the measure of each angle.
   a. \( m \angle AOB = 50^\circ \)
   b. \( m \angle BOC = \) _____
   c. \( m \angle AOC = \) _____
   d. \( m \angle EOD = \) _____
   e. \( m \angle BOD = \) _____
   f. \( m \angle BOE = \) _____
8. Use a protractor to determine the measure of each angle.

![Diagram of angles]

\[ a. \ m\angle TQP = \quad b. \ m\angle TQR = \quad c. \ m\angle RQP = \]

9. Using your results from Items 7 and 8, describe any patterns that you notice.

10. Given that point \( D \) is in the interior of \( \angle ABC \), describe how to determine the measure of \( \angle ABC \), without measuring, if you are given the measures of \( \angle ABD \) and \( \angle DBC \).

11. Use the angle addition postulate and the given information to complete each statement.

\[ a. \text{ If } P \text{ is in the interior of } \angle XYZ, m\angle XYP = 25^\circ, \text{ and } m\angle PYZ = 50^\circ, \text{ then } m\angle XYZ = \quad. \]

\[ b. \text{ If } M \text{ is in the interior of } \angle RTD, m\angle RTM = 40^\circ, \text{ and } m\angle RTD = 65^\circ, \text{ then } m\angle MTD = \quad. \]

\[ c. \text{ If } H \text{ is in the interior of } \angle EFG, m\angle EFH = 75^\circ, \text{ and } m\angle HFG = (10x)^\circ, \text{ and } m\angle EFG = (20x - 5)^\circ, \text{ then } x = \quad \text{ and } m\angle HFG = \quad. \]
11d. Lines $DB$ and $EC$ intersect at point $F$. If $m\angle BFC = 44^\circ$ and $m\angle AFB = 61^\circ$, then

\[ m\angle AFC = \quad \] 
\[ m\angle AFE = \quad \] 
\[ m\angle EFD = \quad \]

The bisector of an angle is a ray that divides the angle into two congruent adjacent angles. For example, in the figure to the left, if $BD$ bisects $\angle ABC$, then $\angle ABD \cong \angle DBC$.

12. Given: $\overrightarrow{AH}$ bisects $\angle MAT$. Determine the missing measure.

a. $m\angle MAT = 70^\circ$, $m\angle MAH = \quad$

b. $m\angle HAT = 80^\circ$, $m\angle MAT = \quad$

You can use definitions, postulates, theorems, and properties of algebra to determine the measures of angles.

EXAMPLE 3

If $\overrightarrow{QP}$ bisects $\angle DQL$, $m\angle DQP = 5x - 7$ and $m\angle PQL = 11 + 2x$, determine the measure of $\angle DQL$.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$\angle DQP \cong \angle PQL$</td>
<td>1.</td>
<td>Definition of Angle Bisector</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>$m\angle DQP = m\angle PQL$</td>
<td>2.</td>
<td>Definition of congruent angles</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>$5x - 7 = 11 + 2x$</td>
<td>3.</td>
<td>Substitution Property</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>$3x - 7 = 11$</td>
<td>4.</td>
<td>Subtraction Property of Equality</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>$3x = 18$</td>
<td>5.</td>
<td>Addition Property of Equality</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>$x = 6$</td>
<td>6.</td>
<td>Division Property of Equality</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>$m\angle DQP = 5(6) - 7 = 23$</td>
<td>7.</td>
<td>Substitution Property</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>$m\angle PQL = 11 + 2(6) = 23$</td>
<td>8.</td>
<td>Substitution Property</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>$m\angle DQL = m\angle DQP + m\angle PQL$</td>
<td>9.</td>
<td>Angle Addition Postulate</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>$m\angle DQL = 23 + 23 = 46^\circ$</td>
<td>10.</td>
<td>Substitution Property</td>
<td></td>
</tr>
</tbody>
</table>
GUIDED EXAMPLE

Complete the missing statements and reasons.

\(\angle A\) and \(\angle B\) are complementary, \(m\angle A = 3x + 7\), and \(m\angle B = 6x + 11\). Determine the measure of each angle.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (m\angle A + m\angle B = 90^\circ)</td>
<td>1. Definition of (\text{________________} )</td>
</tr>
<tr>
<td>2. (____ + _____ = 90^\circ)</td>
<td>2. Substitution Property</td>
</tr>
<tr>
<td>3. (9x + 18 = 90)</td>
<td>3. Substitution Property</td>
</tr>
<tr>
<td>4. (9x = 72)</td>
<td>4. (\text{________} ) Property of Equality</td>
</tr>
<tr>
<td>5. (x = 8)</td>
<td>5. (\text{________} ) Property of Equality</td>
</tr>
<tr>
<td>6. (m\angle A = 3(8) + 7 = 31)</td>
<td>6. Substitution Property</td>
</tr>
<tr>
<td>7. (m\angle B = 6(____) + 11 = ____)</td>
<td>7. Substitution Property</td>
</tr>
</tbody>
</table>

TRY THESE B

Given: \(FL\) bisects \(\angle AFM\). Determine each missing value.

- \(a.\) \(m\angle LFM = 11x + 4\) and \(m\angle AFL = 12x - 2\)
  
  \(x = ______, m\angle LFM = ______, \) and \(m\angle AFM = ______\)

- \(b.\) \(m\angle AFM = 6x - 2\) and \(m\angle AFL = 4x - 10\)
  
  \(x = ______\) and \(m\angle LFM = ______\)

In this diagram, lines \(AC\) and \(DB\) intersect as shown. Determine the missing values.

- \(c.\) \(x = ______\)
  
  \(m\angle AEB = ______\)
  
  \(m\angle CEB = ______\)

- \(d.\) \(\angle P\) and \(\angle Q\) are complementary. \(m\angle P = 2x + 25\) and \(m\angle Q = 4x + 11\). Determine the measure of each angle.
CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Given: Point $K$ is between points $H$ and $J$, $HK = x - 5$, $KJ = 3x - 12$, and $HJ = 25$.
   Find the value of $x$.
   a. 7   b. 2   c. 6   d. 9

2. If $K$ is the midpoint of $HJ$, $HK = x + 6$, and $HJ = 5x - 6$, then $KJ = ?$
   a. 6   b. 3   c. 12   d. 9

3. Point $D$ is in the interior of $\angle ABC$, $m\angle ABC = 10x - 7$, $m\angle ABD = 6x + 5$, and $m\angle DBC = 36^\circ$.
   What is $m\angle ABD$?
   a. $3^\circ$   b. $23^\circ$   c. $17^\circ$   d. $77^\circ$

4. If $\angle P$ and $\angle Q$ are complementary, $m\angle P = 5x + 3$, and $m\angle Q = x + 3$, then $x = ?$
   a. 30   b. 29   c. 0   d. 14

5. Ray $QS$ bisects $\angle PQR$. If $m\angle PQS = 5x$ and $m\angle RQS = 2x + 6$, then $m\angle PQR = ?$.
   a. $10^\circ$   b. $20^\circ$   c. $6^\circ$   d. $2^\circ$

6. Given: $m\angle 1 = 4x + 30$ and $m\angle 3 = 2x + 48$
   a. $x = \_\_\_\_\_\_$
   b. $m\angle 3 = \_\_\_\_\_\_$
   c. $m\angle 2 = \_\_\_\_\_\_\_$

7. Given: $\overrightarrow{EA} \perp \overrightarrow{ED}$, $EB$ bisects $\angle AEC$, $m\angle AEB = 4x + 1$, and $m\angle CED = 3x$.
   a. $x = \_\_\_\_\_\_$
   b. $m\angle BEC = \_\_\_\_\_\_\_$

8. Given: $m\angle 1 = 2x + 8$, $m\angle 2 = x + 4$, and $m\angle 3 = 3x + 18$

   a. $x = \_\_\_\_\_\_$
   b. $m\angle 4 = \_\_\_\_\_\_$
   c. Is $\angle 4$ complementary to $\angle 2$? Explain.

9. **MATHEMATICAL REFLECTION** How could you model two angles with a common vertex and a common side that are not adjacent angles?